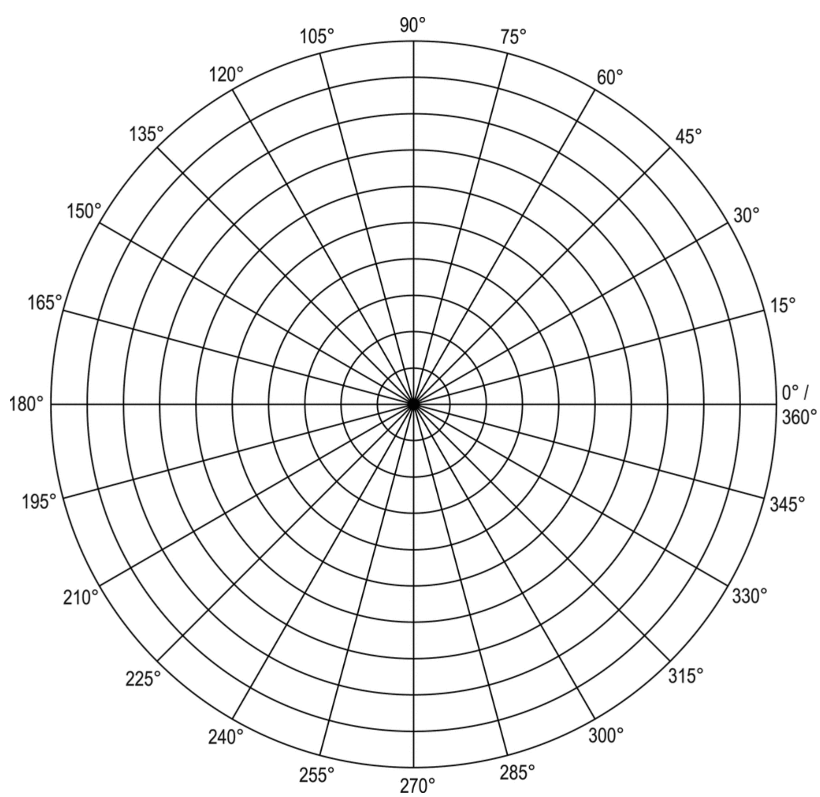
3.16
Polar Coordinate System

Polar Coordinate System – a plane with a point O, the **pole,** and a ray from O, the **polar axis**.

Each point P (r, θ) in the plane is assigned **polar coordinates** as follows: r is the **directed distance** from O to P, and θ is the **directed angle** whose initial side is on the polar axis and whose terminal side is on the line OP.

Measure θ is positive when moving counterclockwise and negative when moving clockwise. If $r > 0$ then P is on the terminal side of θ. If $r < 0$ then P is on the terminal side of $\theta + \pi$.

Examples: P $(2, \pi/3)$ Q $(-1, \pi/4)$ R $(3, -45°)$



Finding all Polar Coordinates of a Point

Let P have polar coordinates (r,θ). Any other polar coordinate of P must be of the form

$$(r,\ \theta + 2n\pi) \text{ or } (-r,\ \theta + (2n + 1)\pi)$$

where n is any integer. In particular the pole has polar coordinates $(0,\ \theta)$, where θ is any angle.

Ex. P = (4, π/5)

Using $(r,\ \theta + 2n\pi)$, then $(4,\ \pi/5 + 2n\pi)$ are also polar coordinates for the same point or location.

Using $(-r,\ \theta + (2n + 1)\pi)$, then $(-4,\ \pi/5 + (2n + 1)\pi)$ are also polar coordinates for the same point or location.

Coordinate Conversion Equations:

Let the point P have polar coordinates $(r,\ \theta)$ and rectangular coordinates $(x,\ y)$. Then

$$x = r\cos\theta \qquad r^2 = x^2 + y^2$$
$$y = r\sin\theta \qquad \tan\theta = y/x.$$

Converting From Polar to Rectangular Coordinates

Examples:

a) P $(3,\ 5\pi/6)$ b) Q $(2,\ -200°)$

a) For P$(3,\ 5\pi/6)$, $r = 3$ and $\theta = 5\pi/6$

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$x = 3\cos\frac{5\pi}{6} \quad \text{and} \quad y = 3\sin\frac{5\pi}{6}$$

$$x = 3\left(-\frac{\sqrt{3}}{2}\right) \approx -2.60 \qquad y = 3\left(\frac{1}{2}\right) = 1.5$$

Now try b.

Converting from Rectangular to Polar Coordinates

Examples:

a) P (-1,1) b) Q (-3, 0)

a) For P(-1, 1), $x = -1$ and $y = 1$

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$

$$r^2 = (-1)^2 + (1)^2 \qquad \tan\theta = \frac{1}{-1} = -1$$

$$r = \pm\sqrt{2} \qquad \theta = \tan^{-1}(-1) + n\pi = -\frac{\pi}{4}$$

Equation Conversion: Polar to rectangular

Example:

$r = 4\cos\theta$ (multiply both sides of the equation by r)

$r^2 = 4r\cos\theta$ (use substitution to rewrite in rectangular form)

$x^2 + y^2 = 4x$

$x^2 - 4x + y^2 = 0$ (complete the square)

$(x - 2)^2 + y^2 = 4$

This is an equation of a circle with center at (2, 0) and radius 2.
Example:

$$r = 4\sec\theta$$

$$\frac{r}{\sec\theta} = 4$$

$$r\cos\theta = 4$$

$$x = 4$$

Rectangular to Polar: $(x - 3)^2 + (y - 2)^2 = 13$

$x^2 - 6x + 9 + y^2 - 4y + 4 = 13$

$x^2 + y^2 - 6x - 4y = 0$

using substitution remember that $r^2 = x^2 + y^2$ and $x = r\cos\theta$ and $y = r\sin\theta$,

so,

$r^2 - 6r\cos\theta - 4r\sin\theta = 0$

$r(r - 6\cos\theta - 4\sin\theta) = 0$

$r = 0$ or $r - 6\cos\theta - 4\sin\theta = 0$

The graph of $r = 0$ consists of a single point at the origin, which is also on the graph of $r - 6\cos\theta - 4\sin\theta = 0$. Therefore, the polar form is:

$$r = 6\cos\theta + 4\sin\theta$$

(Show how to graph polar equations.)

Finding distance using Polar Coordinates ….(remember Law of Cosines)

Ex. Find the distance between two ships if their locations are (5 mi, 78º) and 6 mi, 137º).